Contents

Preface to the second edition v

Preface vi

Index of notation xiii

1 Completely monotone functions 1

2 Stieltjes functions 16

3 Bernstein functions 21

4 Positive and negative definite functions 35

5 A probabilistic intermezzo 48

6 Complete Bernstein functions 69

6.1 Representation of complete Bernstein functions 69

6.2 Extended complete Bernstein functions 80

7 Properties of complete Bernstein functions 92

8 Thorin–Bernstein functions 109

9 A second probabilistic intermezzo 117

10 Transformations of Bernstein functions 131

11 Special Bernstein functions and potentials 159

11.1 Special Bernstein functions 159

11.2 Hirsch's class 172

12 The spectral theorem and operator monotonicity 179

12.1 The spectral theorem 179

12.2 Operator monotone functions 187

13 Subordination and Bochner's functional calculus 200

13.1 Semigroups and subordination in the sense of Bochner 200

13.2 A functional calculus for generators of semigroups 216

13.3 Subordination and functional inequalities 233

13.4 Eigenvalue estimates for subordinate processes 242

14 Potential theory of subordinate killed Brownian motion 257

15 Applications to generalized diffusions 268

15.1 Inverse local time at zero 268

15.2 First passage times 285

16 Examples of complete Bernstein functions 299

16.1 Special functions used in the tables 300

16.2 Algebraic functions 304

16.3 Exponential functions 312

16.4 Logarithmic functions 314

16.5 Inverse trigonometric functions 330

16.6 Hyperbolic functions 330

16.7 Inverse hyperbolic functions 336

16.8 Gamma and related special functions 340

16.9 Bessel functions 350

16.10 Miscellaneous functions 358

16.11 $\mathcal{CBF}$s given by exponential representations 366

16.12 Additional comments 371

Appendix 374

A.1 Vague and weak convergence of measures 374

A.2 Hunt processes and Dirichlet forms 377

Bibliography 383

Index 406